# Prospect Theory and the Demand for Insurance

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#### Abstract

We examine the effect of prospect theory preferences on the demand for insurance to determine whether such preferences can explain the choice of low deductibles observed in the market. Prospect theory implies individuals make decisions by evaluating gains and losses relative to a reference point, where utility is concave over gains and convex over losses; furthermore, losses are weighed more heavily than gains in this setting. We incorporate such preferences in the utility function for an individual and investigate various reference points for an individual making insurance purchasing decisions. We find that prospect theory can explain several phenomena observed in insurance markets: the preference for low deductibles for mandatory insurance, the lack of demand for non-mandatory insurance like catastrophe insurance, and the over-demand to insure small losses as seen with the purchasing of warranties.

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## 1 Introduction

The preference for low deductibles is a well known fact in the insurance literature (Pashigian et al., 1966; Grace et al., 2003; Johnson et al., 1993) that cannot be explained by risk aversion alone. It has been suggested that this preference could be caused by prospect theory preferences (Koszegi Rabin 2006, 2007, 2009), and in this paper, we formally examine whether prospect theory preferences can explain this phenomena. We do so by considering how preferences described by prospect theory can impact the demand for insurance including both the decision to insure and the deductible level chosen. We then compare this deductible level to that chosen by an individual without prospect theory preferences to show how prospect theory can explain the preference for low deductibles.

Cumulative prospect theory, developed by Kahneman and Tversky (1979, 1992) implies individuals make decisions by evaluating gains and losses relative to a reference point rather than evaluating expected final wealth. Prospect theory shows people process these gains/losses using a value function that is concave for gains and convex for losses. This Sshaped value function captures individuals' risk-aversion over gains and risk-seeking behavior over losses. Furthermore, people with prospect theory preferences are willing to take on additional risk in order to avoid feeling a loss. This feature implies individuals weigh losses more heavily than gains, and this aspect of prospect theory has been termed "loss aversion." Finally, prospect theory preferences use a weighting function that overweights small probabilities since individuals have been shown to be more sensitive to small gains/losses relative to larger ones.

In this paper, we investigate the impact of prospect theory preferences on the demand for insurance to see if it can explain the deductible levels observed, especially the preference for low deductibles. Prospect theory induces individuals to take actions to avoid losses and maximize gains. Our intuition is that people will therefore make insurance decisions in order to minimize the domain where a loss is experienced and maximize the domain where a gain is experienced. In this vein, individuals will choose their insurance coverage so as to minimize the experience of a loss should one occur. That is, prospect theory may cause individuals to have a preference for full insurance (or low deductibles).

Previous work that has suggested prospect theory might lead to low deductibles has considered the Koszegi Rabin (2006, 2007, 2009) (KR) framework for prospect theory. The KR framework allows for endogenous reference points; yet applications of this framework to an insurance setting require the elimination of diminishing marginal utility within the framework and assume a linear utility function instead. Braseghyan et al. (2011) implements the KR framework to empirically examine insurance decisions for moderate-stake risks and finds that probability distortions are an important factor in the preference for low deductibles; here probability distortions are with regard to the overweighting of claims probabilities. In the estimation the authors are unable to distinguish between loss aversion and probability weighting, and they do not examine the decision of whether to insure or not, but instead focus on the deductible chosen assuming insurance will be purchased. The authors also assume the loss is always greater than the deductible chosen; therefore they do not necessarily capture how prospect theory preferences may cause the choice for low deductibles initially. Sydnor (2010) looks at insurance purchasing more generally but alludes to prospect theory in the discussion and again, also makes this assumption that the loss experienced is always greater than deductible chosen. Furthermore, both Braseghyan et al. (2011) and Sydnor (2010) make assumptions about the loss distribution. The former uses claims data and assumes it follows a Poisson distribution; the latter assumes losses occur with a discrete probability.

In what follows, we follow the KT approach to model prospect theory which allows for diminishing marginal utility. Eliminating diminishing marginal utility for a framework that examines insurance decisions seems counter-intuitive. Our model accommodates any continuous loss distribution and makes no assumption about the size of the loss relative to the deductible chosen. We consider several benchmarks from which individuals define gains and losses that allows us to distinguish how prospect theory impacts deductible choices when it is assumed insurance will be purchased and when this assumption is not made. That is, we investigate what deductible is chosen when it is known insurance will be purchased, and we also examine situations where individuals decide whether to buy insurance altogether. We find that benchmarks that seem appropriate for insurance that is mandatory will cause those with prospect theory preferences to choose low deductibles. Prospect theory preferences with reference points that seem relevant for non-mandatory insurance can explain the lack of demand for insurance against small probability, high loss events (catastrophe insurance) while also explaining the over-demand to insure small losses (warranties). Initially we examine the loss aversion component of prospect theory and then we incorporate the probability weighting function so as to see how each aspect of prospect theory impacts our results.

In the next section we discuss the previous literature and the two methods for modeling prospect theory (Koszegi-Rabin and Kahneman-Tversky). In section 3, we examine how prospect theory impacts insurance decisions using the Kahneman Tversky framework and investigate how various benchmarks from which individuals can evaluate gains and losses impact insurance decisions. Finally in Section 4, we conclude and discuss future work.

# 2 Prospect Theory Preferences and Previous Literature

Prospect theory was first documented by Kahneman and Tversky (1979) and later examined and quantified further by Tversky and Kahneman (1981, 1992). It implies individuals make decisions by evaluating gains and losses relative to a reference point. The value function to evaluate gains and losses is concave over gains and convex over losses. Furthermore, losses are weighed more heavily than gains. Also probabilities are weighted unevenly with low probabilities being overweighted and moderate/high probabilities being underweighted. Incorporating prospect theory preferences in a model involves the following:

1. Defining a reference point from which to evaluate gains and losses;

2. Invoking a value function that is S-shaped to capture concavity for gains and convexity over losses and that small gains/losses are weighed more heavily than large gains/losses; and

3. Invoking a probability weighting function to capture the overweighting of low probabilities and underweighting of moderate and high probabilities.

Tversky and Kahneman (1992) experimentally tested their idea of cumulative prospect theory and proposed the following functional form for the value function with which to evaluate gains and losses:

$$u(w) = \left\{ \begin{array}{c} w^{a} \text{ if } w \ge 0\\ -\lambda \left(-w\right)^{\beta} \text{ if } w < 0 \end{array} \right\}.$$

$$\tag{1}$$

where w is an outcome, either positive or negative. The authors found that  $a = \beta = .88$ and  $\lambda = 2.25$ . Furthermore, they suggest the following weighting functions to capture the overweighting of small probabilities and underweighting of moderate and high probabilities:

$$w^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}}$$

$$w^{-}(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{1/\delta}}$$
(2)

where p is probability of the outcome and  $w^+$  and  $w^-$  are the decision weights on positive outcomes and negative outcomes respectively. Tversky and Kahneman (1992) find that  $\gamma = .61$  and  $\delta = .69$ . The value function given in (1) and weighting function shown in (2) describe the KT framework for prospect theory.

Previous work in the finance area to examine the effect of prospect theory preferences on financial markets has relied on the framework initially derived by Tversky and Kahneman (1981, 1992) by utilizing the KT framework described here. With this approach, prospect theory has been shown to explain the disposition effect (Barberis and Xiong, 2009), the high mean, excess volatility, and predictability of stock returns (Barberis et al., 2001), and the pricing of a security's own skewness (Barberis and Huang, 2008). Furthermore it has implemented in the pricing of financial derivatives (Pena et al., 2010; Polkovnichenko and Zhao, 2009; Versluis et al., 2010).

Recently, Kosegi and Rabin (2006) have incorporated the concept of prospect theory preferences into a general model of dependent preferences although they do not use the KT framework shown here. Their model evaluates absolute consumption and then has a second attribute which evaluates gains/losses which allows for an endogenous reference point equal to one's recent rational expectations about outcomes. They investigate how such preferences impact willingness to pay for a good (2006), preferences over monetary risk (2007), and intertemporal consumption decisions (2009). The Koszegi-Rabin framework has been implemented by Braseghyan et al. (2011) to examine risk preferences calculated from auto and home insurance deductible choices. Sydnor (2010) uses deductible choices from home insurance to discuss the over-insurance of modest risks and discusses how his work could be adapted to the KR framework.

The KR framework does allow for intertemporal consumption and endogenous reference points. The application of the KR framework to an insurance setting necessitates the loss of curvature of the utility function and therefore eliminates diminishing marginal utility (Sydnor, 2010). Furthermore, implementation of the KR framework for insurance decisions (Braseghyan et al., 2011; Sydnor, 2010) assumes the loss realized is always greater than deductible chosen. This somewhat eliminates the ability to capture the possibility that prospect theory is causing the choice for a low deductible so that the loss incurred is always greater than the deductible. These papers calculate the impact of prospect theory from data on deductible choices and assume specific loss distributions.

Furthermore, Barseghyan et al. (2011) do not examine the decision of whether to insure or not. They assume insurance is bought and examine how prospect theory influences the deducible level chosen after the initial purchasing decision is made. In this way, the authors' model does not apply to all types of insurance. It is not clear that the calculations would be similar if it included catastrophe insurance and/or warranties which are not mandatory types of insurance. Their model is applicable for the data they used for calibration which includes moderate stake risks. Implementation of the KR framework in Barseghyan et al. (2011) also assumes a loss if felt whenever a claim is made and gain is felt whenever one does not make a claim; this method may not truly depict the way individuals evaluate and/or view gains and losses. Even if a claim is not made, if a loss is realized individuals might feel a loss. If individuals benchmark gains and losses relative to their wealth then Barseghyan et al. (2011) does not accurately capture the individual's decision making process.

In their explanation for under-annuitization, Brown et al. (2008) find evidence that individuals tend to view insurance as an investment and simply evaluate the expected gains and/or losses associated when deciding whether to buy a policy. In this manner they do not evaluate absolute consumption and then have a second attribute to evaluate the gains and/or losses relative to a reference point, which is what the KR framework assumes. A model that only evaluates gains/losses as the KT framework does might be more appropriate for an insurance setting then.

In what follows, we implement the KT framework shown above, which has been utilized in the finance area, to examine the impact of prospect theory on insurance decisions. This model maintains diminishing marginal utility in the value function and is general enough to accommodate any continuous loss distribution. Also we do not make an assumption about the loss size relative to the deductible chosen which allows us to better examine the impact of prospect theory preferences on the deducible choice. Furthermore we utilize a reference point which is relevant for when it is not assumed insurance will be purchased. In this way we are able to see how prospect theory influences insurance demand for smaller types of insurance, such as warranties, and also larger types of insurance, such as catastrophe insurance. The manner in which we define gains/losses represents more "status quo loss aversion" rather than the definition for gains and losses used by Barseghyan et al. (2011). We implement several benchmarks to define gains and losses but define them relative to what an individual's previous "status quo" is; this definition is more aligned with that of KT.

A growing literature examines how differences in preferences might affect insurance decisions. Most previous work which considers how psychology impacts insurance decisions focuses on regret and/or disappointment. Braun and Muermann (2004) show how regret impacts the demand for insurance, Muermann et al. (2006) examine how regret impacts portfolio choice for defined contribution pension plans, and Huang et al. (2008) analyze how regret can impact an equilibrium insurance setting. Similarly, Gollier and Muermann (2010) consider a decision-making model where individuals have beliefs which include ex-ante optimism and ex-post disappointment to explain the preference for low deductibles. Shapira and Venezia (2008) show experimentally that individuals are subject to an anchoring effect and anchor their preferences on the prices associated with full insurance policies. As a result, they undervalue partial insurance policies (policies with deductibles) causing them to prefer full insurance and low deductibles. In this paper, we show how prospect theory type preferences can lead to low deductibles for insurance that is seen as mandatory; on the other hand it can also explain the lack of demand to insure small probability, high loss events and the over-demand to insure small losses.

# 3 Impact of Prospect Theory on the Demand for Insurance

Suppose an individual is endowed with initial wealth  $w_0 \ge 0$  and faces a monetary loss L which is described by cumulative distribution function F(L) with F(0) = 0 and  $F(w_0) = 1$ . We denote f(L) as the probability density function for the loss distribution. An insurance company offers indemnity contracts with premiums equal to the expected indemnity plus a proportional loading factor,  $\gamma \ge 0$ . This assumption is consistent with a risk-neutral insurer in a perfectly competitive insurance market with transactional costs but no entry costs. Our setting does not include any information asymmetries that would lead to moral hazard or adverse selection problems.

An insurer offers a set of deductible contracts with deductible levels  $D \in [0, w_0]$ . The indemnity schedule is therefore

$$I(L) = \max(L - D, 0) = (L - D)^{+}$$

and for a given deductible, the premium is given by

$$P(D) = (1 + \gamma) E[I(L)]$$
  
= (1 + \gamma) E[(L - D)<sup>+</sup>].

The individual chooses a deductible level, D, to maximize expected utility of final wealth. We assume individual's preferences can be represented by the KT value function given in (1). For a given benchmark from which to define gains and losses, people choose the level of insurance that maximizes their expected utility; that is they maximize the overall value of gains and losses. For now we focus on how the value function associated with prospect theory impacts insurance decisions. We plan to also incorporate the probability weighting function described by KT to determine how that feature further effects insurance demand.

In what follows, we first note the optimal demand for insurance for an individual not subject to prospect theory preferences. Then we determine the optimal demand for insurance for individuals with prospect theory preferences using three different benchmarks. The first benchmark captures insurance setting where individuals know they will buy insurance, but have to determine how much insurance to purchase. For this setting, individuals define gains and losses relative to initial wealth minus the premium as they assume the premium will be paid and do not factor that into any feeling of a loss. The second benchmark captures both the decision to buy insurance and how much insurance to buy. Here, individuals define gains and losses relative to initial wealth. Finally the third benchmark is an extension of the first benchmark but is state dependent. It is possible that people determine ex-post feelings of losses and gains depending on the outcome that occurred. That is, they implement different benchmarks for different states of the world. For each benchmark we determine the optimal deductible level when individuals maximize expected utility of gains and losses. We denote  $D^*$  as the optimal deductible for an individual with prospect theory preferences.

**Non-Prospect Theory Individual** In addition to understanding how prospect theory influences insurance demand we would like to compare our results with deductibles optimally chosen by an individual not subject to prospect theory. An individual that does not have prospect theory preferences does not evaluate gains and losses. They choose the optimal deductible by maximizing expected utility of final wealth where final wealth is given by

$$W(D) = w_0 - P(D) - L + (L - D)^+$$
  
= w\_0 - P(D) - min(L, D).

We denote  $D_0^*$  as the optimal deductible for an individual that does not have prospect theory preferences. For an increasing, concave utility function, as shown in Mossin (1968), a fully rational, risk-averse individual will buy full insurance ( $D_0^* = 0$ ) if the contract is fairly priced. Partial insurance will be chosen if there is a loading factor ( $\gamma > 0$ ), moral hazard (Holmstrom, 1979), or adverse selection (Rothchild and Stiglitz, 1976).

In order to make the utility for a non-prospect theory individual more comparable to that of a prospect theory individual, we assume they have utility that is increasing and concave and specify it by the following function:

$$u\left(w\right) = w^{a}.\tag{3}$$

The parameter, a, is the same as that from the KT value function. Note that this formula is similar to that for CRRA utility. Additionally, in order for the utility function specified above to be concave, it is necessary that a < 1. This condition is still consistent with KT as they find a = 0.88 (Tversky and Kahneman, 1992). Using the utility given in (3) we find that non-prospect theory individuals will buy full insurance ( $D_0^* = 0$ ) if the contract is fairly priced ( $\gamma = 0$ ) and will demand partial insurance ( $D_0^* > 0$ ) if there is a positive loading factor ( $\gamma > 0$ ). Please see Appendix A.1 for details.

#### 3.1 Benchmark to Initial Wealth Minus Premium

The first benchmark we consider is a reference point equal to initial wealth minus the premium. That is, the decision to buy insurance is already made and individuals assume the premium will be paid but have to decide the optimal deductible level. That is, individuals do not view the premium as a loss and instead think of their starting wealth as the level of wealth which already accounts for paying a premium. One can think of this setting as that associated with insurance that is mandatory, such as auto insurance. The decision to insure has already been made, but people need to decide "how much" insurance to purchase. This decision is influenced by evaluating the expected gains and or losses associated with buying insurance. That is, if they experience a monetary loss which is lower than the deductible chosen, individuals will *feel* a loss. When the monetary loss experienced is greater than the deductible chosen, individuals will *feel* a gain. Our intuition is that individuals will consequently choose lower deductibles in order to minimize the situations in which they feel a loss and maximize situation in which they feel a gain.

The manner in which we define gains and losses for this reference point is as follows: suppose there is no loss. Final wealth equals initial wealth minus the premium  $(w_0 - P(D))$  which implies that final wealth is equal to the individual's benchmark. Therefore the individual does not feel either a gain or a loss. When a loss occurs, the feeling of a gain or loss is dependent on whether the loss incurred is less than or greater than the deductible chosen. If the loss is less than the deductible chosen, then final wealth equals  $(w_0 - P(D) - L)$ . If individuals benchmark to initial wealth minus the premium, they feel a loss equal to the loss incurred, i.e. -L. If the loss incurred is greater than the deductible chosen final wealth equals initial wealth minus the premium and deductible  $(w_0 - P(D) - D)$ . In this case, the individual gains the difference between the loss incurred and the deductible paid; that is, a gain of (L - D) is experienced.

Anticipating these gains and losses, an individual chooses the deductible to maximize his expected utility of gain and losses. The maximization problem is as follows:

$$\max_{D \in [0,w_0]} \left[ \int_{0}^{D} u\left(-L\right) dF\left(L\right) + \int_{D}^{\infty} u\left(L-D\right) dF\left(L\right) \right]$$

which can be reduced to

$$\max_{D\in[0,w_0]} \left[ \int_{0}^{D} -\lambda \left(L\right)^{\beta} dF\left(L\right) + \int_{D}^{\infty} \left(L-D\right)^{a} dF\left(L\right) \right].$$

In the next proposition we show that when individuals do not factor the premium paid into their evaluation of gains and losses, they will always choose full insurance (zero deductible).

**Proposition 1** If a prospect theory individual uses initial wealth minus the premium as his reference point, he will demand full insurance. That is,  $D^* = 0$  for all  $\gamma$ .

#### **Proof.** See Appendix A.2.

When an individual with prospect theory preferences uses initial wealth minus the premium as his reference point, full insurance is always optimal, even at unfair prices ( $\gamma > 0$ ). If full insurance is not offered, then within the contracts offered by the insurer, the individual will choose the policy that provides the most coverage. When insurance is mandatory it is possible that people might "write off" the premium knowing that it will have to be paid. Therefore, gains and losses will be evaluated relative to initial wealth minus the premium paid. If a loss occurs and the loss size is less than the deductible, the individual will feel a loss. However, if the loss size is greater than the deductible chosen a gain will be felt. In order to maximize the number of situations in which a gain is experienced, the individual will choose the lowest deductible available. In the next proposition, we show that an individual with prospect theory preferences will optimally always demand more insurance than a non-prospect theory individual.

**Proposition 2** If a prospect theory individual uses initial wealth minus the premium as his reference point, he will demand a higher level of insurance than a non-PT individual (i.e.  $D^* < D_0^*$ ) for all  $\gamma$ .

#### **Proof.** See Appendix A.3.

Without prospect theory preferences, individuals demand full insurance for actuarially fair premiums ( $\gamma = 0$ ) and partial insurance for unfair premiums ( $\gamma > 0$ ) (Mossin, 1968). With prospect theory preferences, if individuals use initial wealth minus the premium as a benchmark to evaluate gains and losses, an insured only experiences a gain when the loss incurred is greater than the deductible chosen. Consequently a prospect theory individual will demand full insurance ( $D^* = 0$ ) even for unfair premiums to maximize feeling a gain. Both results above demonstrate that individuals with prospect theory type preferences optimally select the lowest deductible available when using initial wealth minus the premium as their reference point. This benchmark seems applicable to mandatory insurance as insureds know the premium will need to paid. Therefore, the results here can help explain the preference for low deductibles that has been documented in the literature both empirically for auto insurance and also through experiments (Pashigian et al., 1966; Grace et al., 2003; Johnson et al., 1993).

### 3.2 Benchmark to Initial Wealth

The second benchmark we consider is a reference point equal to initial wealth. This reference point does not assume the individual writes off the premium as in the previous section. This benchmark would applicable for non-mandatory insurance where the individual would potentially not want to buy the insurance altogether and needs to evaluate whether paying the premium is worthwhile. With this reference point, the gain experienced would need to be enough so that insurance "pays off"; that is, the gain felt needs to offset the premium paid and the loss or deductible, should a loss occur. By benchmarking to initial wealth, the individual needs to decide whether it is "worth it" to purchase insurance initially; if so, then he needs to decide how much insurance to buy.

In this setting, gains and losses are defined as follows. If no loss occurs, final wealth equals initial wealth minus the premium  $(w_0 - P(D))$  and the individual feels a loss equal to the premium paid since the insurance did not "pay off." If a loss occurs but it is lower than the deductible level, then the individual feels a loss of both the premium and the loss that occurred (final wealth would be  $w_0 - P(D) - L$  in this case). If the loss incurred is greater than the deductible then final wealth is given by:  $w_0 - P(D) - D$ . If the amount of the loss above the deductible is less than the premium paid (L - D < P(D)), then the individual will still feel the insurance didn't "pay off" and will feel a loss equal to the premium plus deductible minus the loss (i.e. loss of P(D) + D - L). To understand this situation better consider the following example. Suppose the premium paid is \$500, the deductible is \$1000 and the loss incurred is \$1100. On net, the individual paid \$1500 (premium plus deductible). If the individual had not bought insurance, he would have incurred the \$1100 loss. In this way, the loss felt is the difference between being insured and not (-\$1500 vs. -\$1100); the loss felt is \$400 which is equal to the premium plus deductible minus the loss (1000 + 500-\$1100 = \$400). If the loss is greater than the deductible chosen and the amount of the loss above the deductible is greater than the premium paid (L - D > P(D)) then the individual feels a gain equal to how much the loss offset the costs of buying insurance; that is a gain of L - (P(D) + D) is felt.

Anticipating these gains and losses, an individual chooses the deductible to maximize his expected utility of gains and losses. The maximization problem is as follows:

$$\max_{D \in [0,w_0]} \begin{bmatrix} \int_{0}^{D} u(-P(D) - L)dF(L) + \int_{D}^{D+P(D)} u(-P(D) - D + L)dF(L) \\ + \int_{D+P(D)}^{\infty} u(L - P(D) - D)dF(L) \end{bmatrix}$$

which substituting in the utility from KT as given in (1) we can write as

$$\max_{D \in [0,w_0]} \left[ \begin{array}{c} \int \\ \int \\ -\lambda (P(D) + L)^{\beta} dF(L) + \int \\ \int \\ D \\ -\lambda (P(D) + D - L)^{\beta} dF(L) \\ + \int \\ D \\ D \\ -P(D) \end{array} (L - P(D) - D)^{a} dF(L) \right]$$

In the next proposition we show that for actuarially fair premiums, individuals with prospect theory preferences using initial wealth as a benchmark demand full insurance. When premiums are actuarially unfair, partial insurance is demanded. However, for loss distributions that are weighted heavily toward higher losses, individuals with prospect theory type preferences will choose a lower deductible than non-PT individuals. For loss distributions that weighted toward lower losses, prospect theory individuals choose a higher deductible.

**Proposition 3** If a prospect theory individual uses initial wealth as his reference point, he will demand full insurance if premiums are actuarially fair and partial insurance if premiums are actuarially unfair. That is,  $D^* = 0$  if  $\gamma = 0$  and  $D^* > 0$  if  $\gamma > 0$ . Furthermore, if the following holds, then prospect theory individuals will demand less insurance than a non-prospect theory individual  $(D^* > D_0^*)$ :

$$\int_{0}^{D_{0}^{*}} \beta(P(D_{0}^{*})+L)^{\beta-1}f(L) dL \qquad (4)$$

$$> \int_{D_{0}^{*}+P(D_{0}^{*})} \beta(P(D_{0}^{*})+D_{0}^{*}-L)^{\beta-1}f(L) dL + \int_{D_{0}^{*}+P(D_{0}^{*})}^{\infty} a(L-P(D_{0}^{*})-D_{0}^{*})^{a-1}f(L) dL.$$

#### **Proof.** See Appendix A.4. ■

When the reference point is given as initial wealth, insurance is seen more as an investment. A gain is only felt if the loss is greater than both the premium and the deductible. In this scenario, the result is not as straightforward as it was with the previous benchmark. Overall, individuals with prospect theory preferences will demand full insurance if prices are actuarially fair, just as non-PT individuals do. Yet it's possible there are situations where a prospect theory individual will demand more or less insurance than a non-PT individual. Condition (4) holds when the probability distribution for losses is skewed left. Therefore, for small losses that occur with a high probability prospect theory individuals will be less likely than non-PT individuals to buy insurance and will optimally choose a higher deductible. In this instance, the chance that a gain will be felt is small and hence less insurance is purchased. Catastrophic events have small probabilities associated with high loss sizes and therefore may fall into this instance. In this way, prospect theory may explain the lack of demand for catastrophe insurance. For loss distributions that are not skewed left though, there is a greater chance a gain will be felt, and therefore prospect theory can lead to a preference for more insurance and lower deductibles. Individuals may feel that warranties are more likely to "pay off" and therefore are more willing to buy this type of insurance (at an unfair rate).

### 3.3 State-Dependent Benchmark

which can be reduced to

The last benchmark we consider is one that depends on the state that occurs. In this scenario, we assume the decision to buy insurance is already made and only the deductible level needs to be chosen. Individuals will assume the premium will be paid and do not consider this in their evaluation of gains and losses. They will benchmark their view of gains and losses relative to initial wealth minus the premium for small losses (losses less than the deductible). For large losses that exceed the deductible, however, it is possible insureds feel satisfied that they do not have to incur the loss. However, they did have to pay the deductible so they might not feel a gain either. That is, they feel neither a loss of the deductible nor a gain of how much the loss exceeded the deductible. In this setting if no loss occurs, final wealth equals initial wealth minus the premium and therefore neither a gain nor a loss is felt. If a loss occurs that is less than the deductible chosen, a loss equal to the size of the loss incurred will be felt. If the loss incurred is greater than the deductible, then the individuals feel neither a gain nor a loss since they feel the insurance was worth it.

For this setting, the individual chooses the deducible to maximize expected utility of gains and losses as given by:

$$\max_{D \in [0,w_0]} \left[ \int_{0}^{D} u(-L) dF(L) \right]$$
$$\max_{D \in [0,w_0]} \left[ \int_{0}^{D} -\lambda (L)^{\beta} dF(L) \right]$$

In the following proposition we show it is optimal for the prospect theory individual to choose full insurance.

**Proposition 4** If a prospect theory individual has a state-dependent reference point, he will demand full insurance. That is,  $D^* = 0$  for all  $\gamma$ .

#### **Proof.** See Appendix A.5. ■

When using initial wealth minus the premium as the benchmark for small losses only, a loss is felt as long as the loss incurred is less than the deductible. Once a loss is greater than the deductible, the individual feels the insurance was worth it and does not feel a loss. At the same time, if the loss exceeds the deductible, the individual does have to pay the deductible and so a gain is not felt either. In order to minimize the feeling of a loss, the individual chooses the lowest deductible available to maximize the chance that the loss will be greater than the deductible. In this way, prospect theory supports the preference for low deductibles that has been observed (Pashigian et al., 1966; Grace et al., 2003; Johnson et al., 1993).

## 4 Conclusion and Future Work

We examine the effect of prospect theory preferences on the demand for insurance to determine whether prospect theory can explain the preference for low deductibles as suggested by Sydnor (2010). Prospect theory implies individuals make decisions by evaluating gains and losses relative to a reference point, where utility is concave over gains and convex over losses; furthermore, losses are weighed more heavily than gains in this setting. We incorporate such preferences in the utility function for an individual and investigate various reference points for an individual making insurance purchasing decisions. We find that prospect theory can explain several documented phenomena about deductible choices: the preference for low deductibles for mandatory insurance, the lack of demand for non-mandatory insurance like catastrophe insurance, and the over-demand to insure small losses as seen with the purchasing of warranties.

This work provides additional insight into how consumers behave which has implications for insurance companies on how to better induce individuals to buy coverage. Future work includes investigating how prospect theory impacts an individual's willingness to pay for a reduction in the risk (risk premium). If prospect theory causes people to demand more insurance, then their risk premium should be higher. Additionally, our model does not currently include the decision weights as given by KT which captures people's increased sensitivity to low probability gains/losses relative to medium or large gains/losses. We would like to incorporate this feature to see how it further impacts our results. We also plan to consider the impact of asymmetric information through moral hazard. People subject to prospect theory feel a gain (i.e. insurance "pays off") when the loss incurred is greater than the deductible. The greater the loss, the greater the gain that is felt then. It is possible that after the time of contracting, individuals may alter their behavior so as to increase the probability and/or size of a loss, and hence increase the gain experienced. We would like to investigate the impact of such behavior as it might have implications for the type of contract insurers should offer.

## A Appendix: Proofs and Derivations

### A.1 Proof of Optimal Demand for Non-PT Individual

The maximization problem for a non-PT investor is given by

$$\max_{D_0 \in [0,w_0]} \left[ \int_{0}^{D_0} u(w_0 - P(D_0) - L) dF(L) + \int_{D_0}^{\infty} u(w_0 - P(D_0) - D_0) dF(L) \right]$$

where utility is given by

$$u(w) = w^a$$
.

The maximization problem is therefore reduced to

$$\max_{D_0 \in [0,w_0]} \left[ \int_0^{D_0} (w_0 - P(D_0) - L)^a dF(L) + \int_{D_0}^\infty (w_0 - P(D_0) - D_0)^a dF(L) \right]$$
  
= 
$$\max_{D_0 \in [0,w_0]} \left[ \int_0^{D_0} (w_0 - P(D_0) - L)^a dF(L) + (w_0 - P(D_0) - D_0)^a (1 - F(D_0)) \right].$$

Using Liebnitz' Rule the first derivative is

$$\frac{dEU_0}{dD_0} = (w_0 - P(D_0) - D_0)^a f(D_0) + \int_0^{D_0} a(w_0 - P(D_0) - L)^{a-1} (-P'(D_0)) dF(L) 
+ (w_0 - P(D_0) - D_0)^a (-f(D_0)) + (1 - F(D_0)) a(w_0 - P(D_0) - D_0)^{a-1} (-P'(D_0) - 1) 
= -P'(D_0) \int_0^{D_0} a(w_0 - P(D_0) - L)^{a-1} dF(L) - (P'(D_0) + 1) a(w_0 - P(D_0) - D_0)^{a-1} (1 - F(D_0))$$

Using Liebnitz' Rule again, the second derivative is

$$\frac{d^{2}EU_{0}}{dD_{0}^{2}} = -P'(D_{0})a(w_{0} - P(D_{0}) - D_{0})^{a-1} + (P'(D_{0}))^{2} \int_{0}^{D_{0}} a(a-1)(w_{0} - P(D_{0}) - L)^{a-2}dF(L) 
-P''(D_{0}) \int_{0}^{D_{0}} a(w_{0} - P(D_{0}) - L)^{a-1}dF(L) 
- (P'(D_{0}) + 1) \begin{bmatrix} -f(D_{0})a(w_{0} - P(D_{0}) - D_{0})^{a-1} \\ -(1 - F(D))a(a-1)(w_{0} - P(D_{0}) - D)^{a-2}P'(D_{0}) \end{bmatrix} 
-a(w_{0} - P(D_{0}) - D_{0})^{a-1}P''(D_{0})(1 - F(D))$$

Mossin (1968) showed that the second derivative for the above problem was less than zero if utility is increasing and concave. Note that for this problem,  $u'(w) = aw^{a-1}$  and  $u''(w) = a(a-1)w^{a-2}$ ; therefore utility is increasing and concave for a < 1 which we assume to be consistent with Kahneman and Tversky. Hence  $\frac{d^2 E U_0}{dD_0^2} < 0$  which implies the solution to  $\frac{dE U_0}{dD_0} = 0$  is a global

maximum.

Evaluating the first derivative at the full insurance point  $(D_0 = 0)$  we find

$$\frac{dEU_0}{dD_0}|_{D_0=0} = -\left(P'(0)+1\right)a(w_0 - P(0))^{a-1}\left(1 - F(0)\right)$$
$$= \gamma a(w_0 - P(0))^{a-1}$$

Define  $D_0^*$  as the optimal deductible for a non-PT individual which satisfies  $\frac{dEU_0}{dD_0}|_{D_0^*} = 0$ . If insurance is actuarially fair ( $\gamma = 0$ ) then  $\frac{dEU_0}{dD_0}|_{D_0=0} = 0$  and full insurance is optimal ( $D_0^* = 0$ ). If insurance has a positive loading ( $\gamma > 0$ ) then  $\frac{dEU_0}{dD_0}|_{D_0=0} > 0$  and partial insurance is optimal ( $D_0^* = 0$ ). ( $D_0^* > 0$ ).

## A.2 Proof of Proposition 1

Individual chooses deductible to maximize expected utility of gain/loss

$$\max_{D \in [0,w_0]} \left[ \int_0^D u\left(-L\right) dF\left(L\right) + \int_D^\infty u\left(L-D\right) dF\left(L\right) \right]$$

which can be reduced as follows

$$\max_{D \in [0,w_0]} \left[ \int_{0}^{D} -\lambda (L)^{\beta} dF(L) + \int_{D}^{\infty} (L-D)^{a} dF(L) \right]$$
$$\max_{D \in [0,w_0]} \left[ -\frac{\lambda}{\beta+1} (F(D))^{\beta+1} + \int_{D}^{\infty} (L-D)^{a} dF(L) \right]$$

Using Liebnitz Rule the first derivative is

$$\frac{dEU}{dD} = -\lambda \left(F\left(D\right)\right)^{\beta} f\left(D\right) - \int_{D}^{\infty} a \left(L - D\right)^{a-1} dF\left(L\right)$$

Evaluating the first derivative at zero we find:

$$\frac{dEU}{dD} \quad | \quad _{D=0} = -\lambda \left(F(0)\right)^{\beta} f(0) - \int_{0}^{\infty} a\left(L\right)^{a-1} dF(L)$$
$$= -\int_{0}^{\infty} a\left(L\right)^{a-1} dF(L)$$
$$< 0.$$

The above implies  $D^* < 0$ . As over-insurance isn't allowed, then full insurance  $(D^* < 0)$  is optimal for all loading factors. Using Liebnitz Rule again, the second derivative is given by

$$\frac{d^{2}EU}{dD^{2}} = -\lambda \left(F\left(D\right)\right)^{\beta} f'\left(D\right) - \lambda\beta \left(f\left(D\right)\right)^{2} \left(F\left(D\right)\right)^{\beta-1} + \int_{D}^{\infty} a\left(a-1\right) \left(L-D\right)^{a-2} dF\left(L\right)$$
$$= -\lambda \left(F\left(D\right)\right)^{\beta} \left[f''\left(D\right) + \left(f\left(D\right)\right)^{2} F\left(D\right)^{-1}\right] + \int_{D}^{\infty} a\left(a-1\right) \left(L-D\right)^{a-2} dF\left(L\right)$$

Evaluate at zero as

$$\frac{d^2 E U}{dD^2} \quad | \quad _{D=0} = -\lambda \left( F\left(0\right) \right)^{\beta} \left[ f''\left(0\right) + \left(f\left(0\right)\right)^2 F\left(0\right)^{-1} \right] + \int_0^\infty a\left(a-1\right) \left(L\right)^{a-2} dF\left(L\right) \\ = \int_0^\infty a\left(a-1\right) \left(L\right)^{a-2} dF\left(L\right)$$

Since a < 1 then  $\frac{d^2 EU}{dD^2} \mid_{D=0} < 0$ .

## A.3 Proof of Proposition 2

Consider the first derivative for the prospect theory individual:

$$\frac{dEU}{dD} = -\lambda \left(F\left(D\right)\right)^{\beta} f\left(D\right) - \int_{D}^{\infty} a \left(L - D\right)^{a-1} dF\left(L\right).$$

From the first order condition for the non-PT individual we know

$$\frac{dEU_0}{dD_0} \mid_{D_0 = D_0^*} = \left( \begin{array}{c} D_0^* \\ -P'(D_0^*) \int a(w_0 - P(D_0^*) - L)^{a-1} dF(L) \\ 0 \\ -(P'(D_0^*) + 1) a(w_0 - P(D_0^*) - D_0^*)^{a-1} (1 - F(D_0^*)) \end{array} \right) = 0$$

which implies

$$\begin{aligned} -P'(D_0^*) \int_0^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) &= \left(P'(D_0^*) + 1\right) a(w_0 - P(D_0^*) - D_0^*)^{a-1} (1 - F(D_0^*)) \\ &1 - F(D_0^*) &= -\frac{1}{a} (w_0 - P(D_0^*) - D_0^*)^{a-2} \left(\frac{P'(D_0^*)}{P'(D_0^*) + 1}\right) \\ &\times \int_0^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) \\ &1 - F(D_0^*) &= -\frac{1}{a} (w_0 - P(D_0^*) - D_0^*)^{a-2} \left(\frac{-(1 + \gamma)(1 - F(D_0^*))}{-(1 + \gamma)(1 - F(D_0^*)) + 1}\right) \\ &\times \int_0^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) \end{aligned}$$

$$(1 - F(D_0^*)) \left(\frac{-(1 + \gamma)(1 - F(D_0^*)) + 1}{-(1 + \gamma)(1 - F(D_0^*))}\right) &= -\frac{1}{a} (w_0 - P(D_0^*) - D_0^*)^{a-2} \int_0^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) \\ (1 - F(D_0^*)) \left(\frac{-(1 + \gamma)(1 - F(D_0^*)) + 1}{-(1 + \gamma)(1 - F(D_0^*))}\right) &= -\frac{1}{a} (w_0 - P(D_0^*) - D_0^*)^{a-2} \int_0^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) \\ F(D_0^*) &= \frac{\gamma}{1 + \gamma} + \frac{1}{a} (w_0 - P(D_0^*) - D_0^*)^{a-2} \\ &\times \int_0^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) \end{aligned}$$

Evaluate at the first derivative for the prospect theory individual at the optimal deductible for a non-PT individual as

$$\frac{dEU}{dD} \mid D = D_0^* = -\lambda \left( F\left(D_0^*\right) \right)^{\beta} f\left(D_0^*\right) - \int_{D_0^*}^{\infty} a \left(L - D_0^*\right)^{a-1} dF\left(L\right) \\
= -\lambda f\left(D_0^*\right) \left( \frac{\gamma}{1+\gamma} + \frac{1}{a} (w_0 - P(D_0^*) - D_0^*)^{a-2} \int_0^{D_0^*} a (w_0 - P(D_0^*) - L)^{a-1} dF\left(L\right) \right)^{\beta} \\
- \int_{D_0^*}^{\infty} a \left(L - D_0^*\right)^{a-1} dF\left(L\right) \\
< 0.$$

Since  $\frac{dEU}{dD} \mid_{D=D_0^*} < 0$  that implies that  $D^* < D_0^*$  for all  $\gamma$ .

# A.4 Proof of Proposition 3

Maximization problem is given by

$$\max_{D \in [0,w_0]} \left[ \int_{0}^{D} -\lambda (P(D) + L)^{\beta} dF(L) + \int_{D}^{D+P(D)} -\lambda (P(D) + D - L)^{\beta} dF(L) + \int_{D+P(D)}^{\infty} (L - P(D) - D)^{a} dF(L) \right].$$

Since choice variable, D, is in both the limit and term being integrated for each part of the above equation we use Leibnitz' rule to find the FOC which is given by:

$$-\lambda (P(D) + D)^{\beta} f(D) + P'(D) \int_{0}^{D} -\lambda \beta (P(D) + L)^{\beta - 1} dF(L)$$
$$+\lambda (P(D))^{\beta} f(D) + (1 + P'(D)) \int_{D}^{D + P(D)} -\lambda \beta (P(D) + D - L)^{\beta - 1} dF(L)$$
$$- (1 + P'(D)) \int_{D + P(D)}^{\infty} a (L - P(D) - D)^{a - 1} dF(L)$$

and reduces to

$$\frac{dEU}{dD} = -\lambda (P(D) + D)^{\beta} f(D) + \lambda (P(D))^{\beta} f(D) - \lambda \beta P'(D) \int_{0}^{D} (P(D) + L)^{\beta - 1} dF(L)$$
$$-\lambda \beta (1 + P'(D)) \int_{D}^{D+P(D)} (P(D) + D - L)^{\beta - 1} dF(L)$$
$$-a (1 + P'(D)) \int_{D+P(D)}^{\infty} (L - P(D) - D)^{a - 1} dF(L)$$

Recall

$$P(D) = (1 + \gamma) E [(L - D)^+]$$
$$= (1 + \gamma) \int_D^\infty (L - D) dF(L)$$

which implies

$$P'(D) = \frac{d}{dL} (1+\gamma) \int_{D}^{\infty} (L-D) dF(L)$$
$$= -(1+\gamma) (1-F(D)).$$

Evaluating the FOC at D = 0 we find

$$\frac{dEU}{dD} \mid D=0 = -\lambda(P(0))^{\beta} f(0) + \lambda (P(0))^{\beta} f(0) - \lambda\beta (1+P'(0)) \int_{0}^{P(0)} (P(0)-L)^{\beta-1} dF(L) 
-a (1+P'(0)) \int_{P(0)}^{\infty} (L-P(0))^{a-1} dF(L) 
= \lambda\beta\gamma \int_{0}^{P(0)} (P(0)-L)^{\beta-1} dF(L) + a\gamma \int_{P(0)}^{\infty} (L-P(0))^{a-1} dF(L) 
= \gamma \left(\lambda\beta \int_{0}^{P(0)} (P(0)-L)^{\beta-1} dF(L) + a \int_{P(0)}^{\infty} (L-P(0))^{a-1} dF(L)\right)$$

since  $\mathbf{s}$ 

$$P(0) = (1 + \gamma) E[L] P'(0) = -(1 + \gamma).$$

For an actuarially fair premium  $(\gamma = 0)$  we find

$$\frac{dEU}{dD}\mid_{D=0}=0$$

This result implies individuals will choose full insurance  $(D^* = 0)$ .

For a positive loading factor  $(\gamma > 0)$ ,

$$\frac{dEU}{dD}|_{D=0} = \gamma \left( \lambda \int_{0}^{P(0)} \beta \left( P(0) - L \right)^{\beta - 1} dF(L) + \int_{P(0)}^{\infty} a \left( L - P(0) \right)^{a - 1} dF(L) \right) > 0$$

which implies that partial insurance is optimal  $(D^* > 0)$ .

To compare the deductible chosen by a PT individual relative to that chosen by a non-PT individual, evaluate the FOC for a PT individual at the optimal deductible for a non-PT individual

as:

$$\begin{split} \frac{dEU}{dD}|_{D=D_{0}^{*}} &= -\lambda (P(D_{0}^{*}) + D_{0}^{*})^{\beta} f\left(D_{0}^{*}\right) + \lambda \left(P\left(D_{0}^{*}\right)\right)^{\beta} f\left(D_{0}^{*}\right) - \left(\lambda \beta P'\left(D_{0}^{*}\right) \int_{0}^{D_{0}^{*}} (P(D_{0}^{*}) + L)^{\beta-1} dF\left(L\right)\right) \\ &- \left(\lambda \beta \left(1 + P'\left(D_{0}^{*}\right)\right) \int_{D_{0}^{*}}^{\infty} \left(P\left(D_{0}^{*}\right) + D_{0}^{*} - L\right)^{\beta-1} dF\left(L\right)\right) \\ &- \left(\left(1 + P'\left(D_{0}^{*}\right)\right) \int_{D_{0}^{*} + P\left(D_{0}^{*}\right)}^{\infty} a\left(L - P\left(D_{0}^{*}\right) - D_{0}^{*}\right)^{a-1} dF\left(L\right)\right) \\ &= \lambda f\left(D_{0}^{*}\right) \left[ \left(P\left(D_{0}^{*}\right)\right)^{\beta} - \left(P\left(D_{0}^{*}\right) + D_{0}^{*}\right)^{\beta} \right] \\ &- \lambda P'\left(D_{0}^{*}\right) \int_{0}^{D_{0}^{*}} \beta \left(P\left(D_{0}^{*}\right) + L\right)^{\beta-1} dF\left(L\right) \\ &- \lambda \left(1 + P'\left(D_{0}^{*}\right)\right) \int_{D_{0}^{*}}^{\infty} \beta \left(P\left(D_{0}^{*}\right) - D_{0}^{*}\right)^{a-1} dF\left(L\right) \\ &- \left(1 + P'\left(D_{0}^{*}\right)\right) \int_{D_{0}^{*} + P\left(D_{0}^{*}\right)}^{\infty} a\left(L - P\left(D_{0}^{*}\right) - D_{0}^{*}\right)^{a-1} dF\left(L\right). \end{split}$$

The integrals in the last 3 terms are all positive. The first term in the equation above is negative and for  $\gamma > 0$ ,  $D_0^* > 0$  which implies that  $P'(D_0^*) < 0$ . Therefore the 2nd term is positive. To determine the sign of the last three terms consider the FOC for non-PT individual which implies

$$0 = -P'(D_0^*) \int_0^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) - (P'(D_0^*) + 1) a(w_0 - P(D_0^*) - D_0^*)^{a-1} (1 - F(D_0^*)) P'(D_0^*) \int_0^{D_0^*} a(w_0 - P(D_0^*) - L)^{a-1} dF(L) (1 + P'(D_0^*)) = -\frac{0}{a(w_0 - P(D_0^*) - D_0^*)^{a-1} (1 - F(D_0^*))}.$$

We know  $P'(D_0^*) < 0$ , the integral in the term above will be positive as will the denominator which implies that  $(1 + P'(D_0^*)) > 0$ . Going back to the FOC for a PT individual evaluated at  $D_0^*$  we can see that the last two terms will be negative. Therefore all terms except the 2nd term in  $\frac{dEU}{dD}|_{D=D_0^*}$  are negative.

Therefore,  $\frac{dEU}{dD}|_{D=D_0^*} < 0$  unless the 2nd term outweighs. That is unless the following condition

holds:

$$\int_{0}^{D_{0}^{*}} \beta(P(D_{0}^{*})+L)^{\beta-1}dF(L) > \begin{bmatrix} \frac{(1+P'(D_{0}^{*}))}{-P'(D_{0}^{*})} \int_{D_{0}^{*}}^{D_{0}^{*}+P(D_{0}^{*})} \beta(P(D_{0}^{*})+D_{0}^{*}-L)^{\beta-1}dF(L) \\ +\lambda\frac{(1+P'(D_{0}^{*}))}{-P'(D_{0}^{*})} \int_{-P'(D_{0}^{*})}^{\infty} a(L-P(D_{0}^{*})-D_{0}^{*})^{a-1}dF(L) \\ +\frac{f(D_{0}^{*})[(P(D_{0}^{*}))^{\beta}-(P(D_{0}^{*})+D_{0}^{*})^{\beta}]}{P'(D_{0}^{*})} \end{bmatrix}$$

Reduce this term as follows:

$$\int_{0}^{D_{0}^{*}} \beta(P(D_{0}^{*})+L)^{\beta-1} dF(L) > \begin{pmatrix} \frac{(1+P'(D_{0}^{*}))}{-P'(D_{0}^{*})} \int_{D_{0}^{*}}^{\infty} \beta(P(D_{0}^{*})+D_{0}^{*}-L)^{\beta-1} dF(L) \\ +\lambda \frac{(1+P'(D_{0}^{*}))}{-P'(D_{0}^{*})} \int_{D_{0}^{*}+P(D_{0}^{*})}^{\infty} a(L-P(D_{0}^{*})-D_{0}^{*})^{a-1} dF(L) \\ +\frac{f(D_{0}^{*})[(P(D_{0}^{*}))^{\beta}-(P(D_{0}^{*})+D_{0}^{*})^{\beta}]}{P'(D_{0}^{*})} \\ > \begin{pmatrix} D_{0}^{*}+P(D_{0}^{*}) \\ \int_{D_{0}^{*}}^{\infty} \beta(P(D_{0}^{*})+D_{0}^{*}-L)^{\beta-1} dF(L) \\ D_{0}^{*}-P(D_{0}^{*}) \\ +\int_{D_{0}^{*}+P(D_{0}^{*})}^{\infty} a(L-P(D_{0}^{*})-D_{0}^{*})^{a-1} dF(L) \\ D_{0}^{*}+P(D_{0}^{*}) \end{pmatrix} \end{bmatrix}$$

where the last condition holds because

$$\frac{\left(1+P'\left(D_{0}^{*}\right)\right)}{-P'\left(D_{0}^{*}\right)}>1$$

That is, if

$$\sum_{0}^{D_{0}^{*}} \beta(P(D_{0}^{*}) + L)^{\beta - 1} f(L) dL$$

$$> \int_{D_{0}^{*}}^{D_{0}^{*} + P(D_{0}^{*})} \beta(P(D_{0}^{*}) + D_{0}^{*} - L)^{\beta - 1} f(L) dL + \int_{D_{0}^{*} + P(D_{0}^{*})}^{\infty} a(L - P(D_{0}^{*}) - D_{0}^{*})^{a - 1} f(L) dL.$$

then  $\frac{dEU}{dD}|_{D=D_0^*} > 0$  which implies  $D^* > D_0^*$ . This condition would hold if the pdf is weighted heavily toward losses lower than the deductible level. Therefore for loss distributions that are not skewed to the left, the above condition would not hold and we have  $\frac{dEU}{dD}|_{D=D_0^*} < 0$  which implies  $D^* < D_0^*$ .

## A.5 Proof of Proposition 4

Maximization problem is given by

$$\max_{D \in [0,w_0]} \left[ \int_{0}^{D} -\lambda \left( L \right)^{\beta} dF \left( L \right) \right]$$
$$\max_{D \in [0,w_0]} \left[ \left( -\frac{\lambda}{\beta+1} \left( L \right)^{\beta+1} \right) |_{F(0)}^{F(D)} \right]$$
$$\max_{D \in [0,w_0]} \left[ -\frac{\lambda}{\beta+1} \left( F \left( D \right) \right)^{\beta+1} \right]$$

which has a first order condition of

$$\frac{dEU}{dD} = -\lambda \left( F\left( D \right) \right)^{\beta} f\left( D \right) = 0$$

and a SOC of

$$\frac{d^2 E U}{dD^2} = -\lambda \left( F(D) \right)^{\beta} f'(D) - \lambda \beta \left( F(D) \right)^{\beta-1} \left( f(D) \right)^2$$

Most likely pdf is decreasing so 1st term is positive and second term is positive. In order for SOC < 0 we need

$$-F(D) f'(D) < \beta (f(D))^{2}.$$

If condition above holds, SOC < 0 and solution to  $\frac{dEU}{dD} = 0$  is global max. Evaluate FOC at D = 0:

$$\frac{dEU}{dD}|_{D=0} = -\lambda \left(F\left(0\right)\right)^{\beta} f\left(0\right)$$
$$= 0$$

Therefore,  $D^* = 0$ .

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